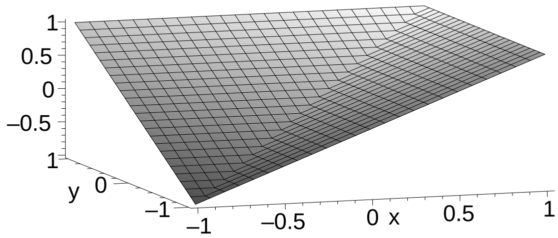
TP 3 : fonctions de plusieurs variables

```
> restart;
```

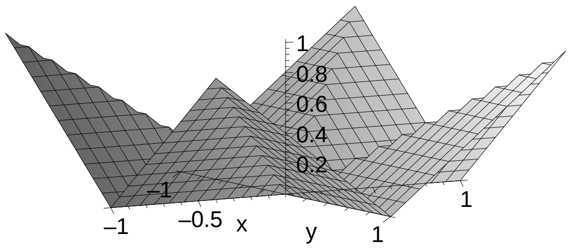
1 Questions de régularité

```
> f1:=(x,y)->max(x,y):f2:=(x,y)->min(abs(x),abs(y)):
> plot3d(f1(x,y),x=-1..1,y=-1..1);
```

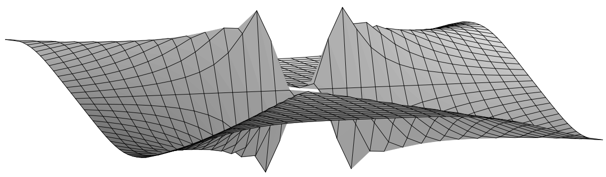


Evidemment à l'impression ,ça ne va pas donner grand chose. Par contre, "à la souris", on peut déplacer la courbe

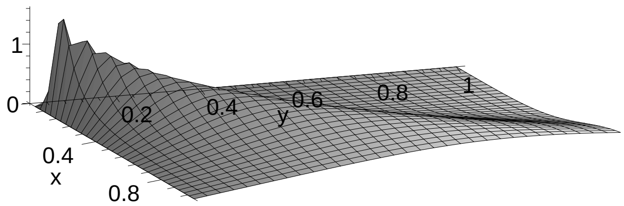
```
> plot3d(f2(x,y),x=-1..1,y=-1..1);
```




```
> f3:=(x,y)->if x=0 and y=0 then 0 else x^4*y/(x^6+y^4) fi:
> plot3d(f3(x,y),x=-1..1,y=-1..1,numpoints=1000);
```



```
> plot3d(f3(x,y),x=0..1,y=0..1,numpoints=1000);
```




```
> f3((rho*cos(theta))^(1/3),(rho*sin(theta))^(1/2));
```

$$\frac{(\rho\cos(\theta))^{(4/3)}\sqrt{\rho\sin(\theta)}}{\rho^2\cos(\theta)^2+\rho^2\sin(\theta)^2}$$

```
> simplify(%);
```

$$\frac{\cos(\theta)\,(\rho\cos(\theta))^{(1/3)}\sqrt{\rho\sin(\theta)}}{\rho}$$

```
> limit(f3(r^(1/3),r^(1/2)),r=0);
```

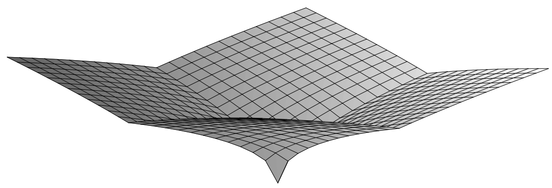
$$\lim_{r \to 0} \frac{1}{2}\frac{1}{r^{(1/6)}}$$

```
> limit(f3(r^(1/3),r^(1/2)),r=0,right);
```

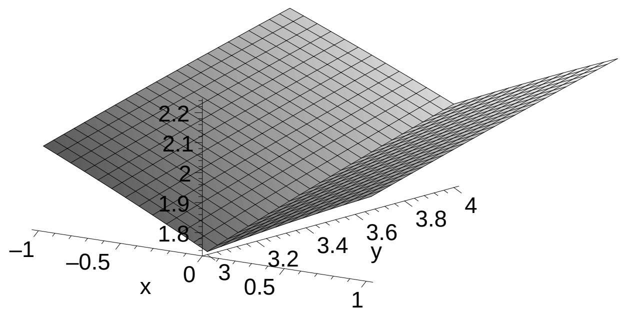
$$\infty$$

f3 est discontinue en 0.

```
> f4:=(x,y)->sqrt(abs(x)+abs(y)):plot3d(f4(x,y),x=-1..1,y=-1..1
  );
```

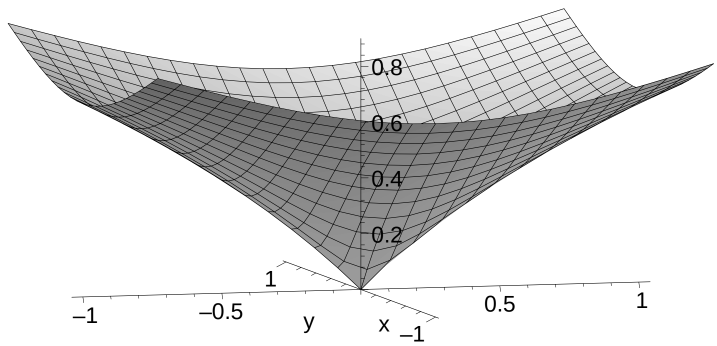


```
> plot3d(f4(x,y),x=-1..1,y=3..4);
```



f4 ne semble pas de classe C1 sur les axes...

```
> f5:=(x,y)->ln(1+sqrt(x^2+y^2)):plot3d(f5(x,y),x=-1..1,y=-1..1
  );
```



f5 semble continue (mais pas de classe C1) en 0.

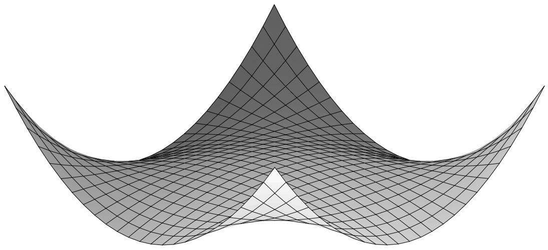
```
> restart;
```

2 Un calcul d’extrema

```
> f:=(x,y)->x^2*y^2-x^2-y^2;
```

$$f := (x, y) \to x^2\, y^2 - x^2 - y^2$$

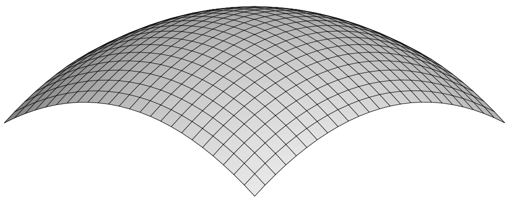
```
> with(plots):
Warning, the name changecoords has been redefined
> plot3d(f(x,y),x=-2..2,y=-2..2);
```



```
> solve({diff(f(x,y),x)=0,diff(f(x,y),y)=0},{x,y});
```

$$\{x = 0, y = 0\}, \{y = 1, x = 1\}, \{y = -1, x = 1\}, \{y = 1, x = -1\}, \{y = -1, x = -1\}$$

```
> plot3d(f(x,y),x=-0.2..0.2,y=-0.2..0.2);
```



Ca ressemble à un maximum local...

```
> f(r*cos(theta),r*sin(theta));
```

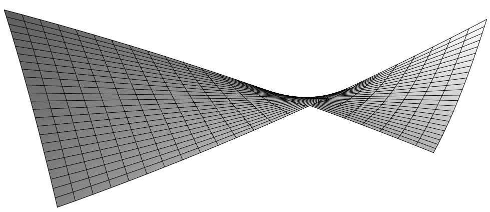
$$r^4 \cos(\theta)^2 \sin(\theta)^2 - r^2 \cos(\theta)^2 - r^2 \sin(\theta)^2$$

```
> simplify(%);
```

$$-r^2 (-r^2 \cos(\theta)^2 + r^2 \cos(\theta)^4 + 1)$$

C’est négatif pout r assez petit...

```
> plot3d(f(x,y),x=0.8..1.2,y=0.8..1.2);
```



Ca ressemble à un point-selle...

```
> expand(f(1+u,1+v));
```

$$-1 + 4\, u\, v + 2\, u\, v^2 + 2\, u^2\, v + u^2\, v^2$$

```
> expand(f(1+u,1+u)),expand(f(1+u,1-u));
```

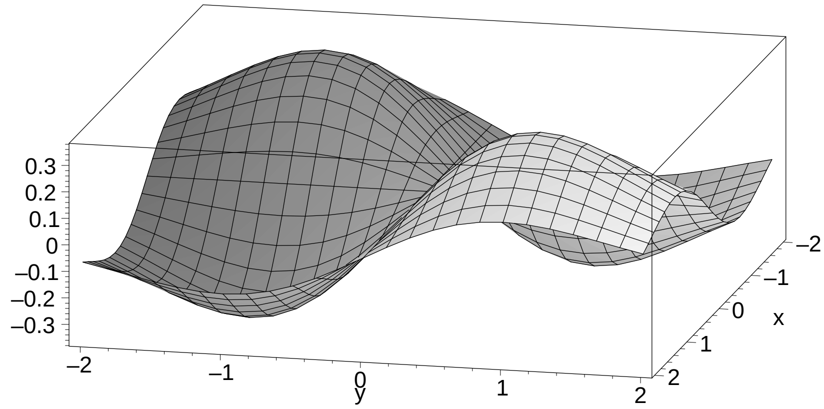
$$-1 + 4\, u^2 + 4\, u^3 + u^4, -1 - 4\, u^2 + u^4$$

Effectivement, dans une direction, on a un minimum local, et dans l’autre un maximum local.

```
> f:=(x,y)->x*y*exp(-(x^2+y^2)/2);
```

$$f := (x, y) \rightarrow x\, y\, \mathbf{e}^{(-1/2\, x^2 - 1/2\, y^2)}$$

```
> plot3d(f(x,y),x=-2..2,y=-2..2);
```



Il semblerait qu'il y ait deux maxima (resp. minima) locaux qui sont des extrema globaux.

```
> solve({diff(f(x,y),x)=0,diff(f(x,y),y)=0});
```

$$\{y = 0, x = 0\}, \{x = 1, y = 1\}, \{x = 1, y = -1\}, \{x = -1, y = 1\}, \{x = -1, y = -1\}$$

```
> f(u,u),f(u,-u);
```

$$u^2\, \mathbf{e}^{(-u^2)}, -u^2\, \mathbf{e}^{(-u^2)}$$

Il y a donc un "point selle" en (0,0)

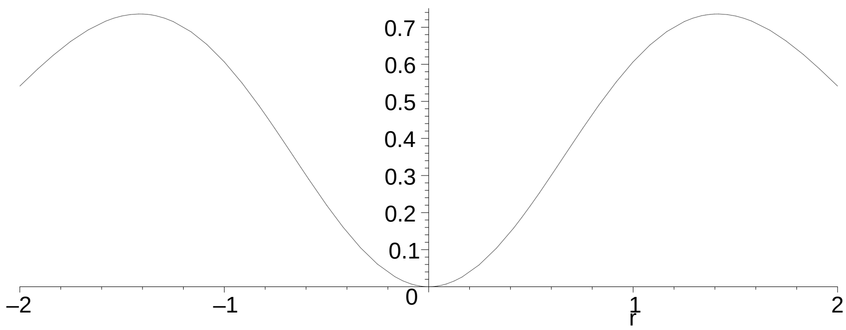
```
> simplify(f(r*cos(theta),r*sin(theta)));
```

$$r^2 \cos(\theta) \sin(\theta)\, \mathbf{e}^{(-1/2\, r^2)}$$

```
> solve(diff(r^2*exp(-r^2/2),r)=0);
```

$$0, \sqrt{2}, -\sqrt{2}$$

```
> plot(r^2*exp(-r^2/2),r=-2..2);
```



On a donc un maximum global (donc local) en les points de coordonnées polaires (Pi/4,sqrt(2)) et (3Pi/4,sqrt(2)), et un minimum global donc local en les points de coordonnées polaires (-Pi/4,sqrt(2)) et (-3Pi/4,sqrt(2)). Il s'agit bien des quatre points critiques trouvés plus haut.

```
> restart;
```

3 Une circulation

```
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
```

```
> A:=[a/3,a/3,a/3];n:=vector([1,1,1])/sqrt(3);f1:=vector([1,-1,
  0])/sqrt(2);f2:=crossprod(n,f1);R:=a*sqrt(2/3);
```

$$A := \left[\frac{1}{3}a, \frac{1}{3}a, \frac{1}{3}a\right]$$

$$n := \frac{1}{3}[1, 1, 1]\sqrt{3}$$

$$f1 := \frac{1}{2}[1, -1, 0]\sqrt{2}$$

$$f2 := \left[\frac{1}{6}\sqrt{3}\sqrt{2}, \frac{1}{6}\sqrt{3}\sqrt{2}, -\frac{1}{3}\sqrt{3}\sqrt{2}\right]$$

$$R := \frac{1}{3}a\sqrt{6}$$

```
> M:=theta->evalm(A+R*(cos(theta)*f1+sin(theta)*f2));
```

$$M := \theta \to \mathrm{evalm}(A + R(\cos(\theta)\, f1 + \sin(\theta)\, f2))$$

```
> M(theta);
```

$$\left[\frac{1}{3}a + \frac{1}{3}a\sqrt{6}\left(\frac{1}{2}\cos(\theta)\sqrt{2} + \frac{1}{6}\sin(\theta)\sqrt{3}\sqrt{2}\right),\right.$$
$$\left.\frac{1}{3}a + \frac{1}{3}a\sqrt{6}\left(-\frac{1}{2}\cos(\theta)\sqrt{2} + \frac{1}{6}\sin(\theta)\sqrt{3}\sqrt{2}\right), \frac{1}{3}a - \frac{1}{9}a\sqrt{6}\sin(\theta)\sqrt{3}\sqrt{2}\right]$$

```
> x:=theta->M(theta)[1]:x(theta);y:=theta->M(theta)[2]:y(theta)
  ;z:=theta->M(theta)[3]:z(theta);
```

$$\frac{1}{3}a + \frac{1}{3}a\sqrt{6}\left(\frac{1}{2}\cos(\theta)\sqrt{2} + \frac{1}{6}\sin(\theta)\sqrt{3}\sqrt{2}\right)$$

$$\frac{1}{3}a + \frac{1}{3}a\sqrt{6}\left(-\frac{1}{2}\cos(\theta)\sqrt{2} + \frac{1}{6}\sin(\theta)\sqrt{3}\sqrt{2}\right)$$

$$\frac{1}{3}a - \frac{1}{9}a\sqrt{6}\sin(\theta)\sqrt{3}\sqrt{2}$$

```
> int(x(theta)*diff(y(theta),theta)+y(theta)*diff(z(theta),thet
  a)+z(theta)*diff(x(theta),theta),theta=0..2*Pi);
```

$$\frac{2}{3}a^2\sqrt{3}\,\pi$$

Ce calcul peut également ^etre fait en deux lignes à l’aide de la ‘‘formule de Stokes’’.